Addressing Positivity Violations in Continuous Interventions through Data-Adaptive Strategies

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Background

Positivity violations challenge the estimation of the causal dose-response curve $m^{\text{standard}} : \mathcal{A} \to \mathcal{Y};\ a \mapsto E(Y^a)$. Existing solutions, such as projection functions [1] or modified treatment policies [2], can address this issue but may yield estimands misaligned with the original research question, reducing interpretability.

Our Contribution

- A novel diagnostic tool—**the non-overlap ratio**—to detect positivity violations for continuous interventions.
- A data-adaptive solution, specifically a **most feasible intervention strategy** to address positivity violations while maintaining interpretability.

Methods

Step 1: Determine positivity violations

A continuous intervention $a \in \mathcal{A} \subseteq \mathbb{R}$ is classified as:

$$\begin{cases} \text{Supported}, & \text{if } a \in \mathcal{A}_\alpha(\mathbf{l}), \\ \text{Not supported}, & \text{otherwise.} \end{cases}$$

Here, $\mathcal{A}_\alpha(\mathbf{l})$ represents the **high-density region** of the intervention space, with α as the **coverage level**, defined as:

$$\mathcal{A}_\alpha(\mathbf{l}) = \{a \in \mathcal{A} : f(a|\mathbf{l}) \geqslant f_\alpha\}, \quad \text{s.t. } P(\mathcal{A}_\alpha(\mathbf{l})) = \alpha.$$

The **non-overlap ratio** quantifies positivity violations as:

$$\tau : \mathcal{A} \to [0,1], \quad a \mapsto \int \mathbb{1}(a \notin \mathcal{A}_\alpha(\mathbf{L}))\, dP_{\mathbf{L}}.$$

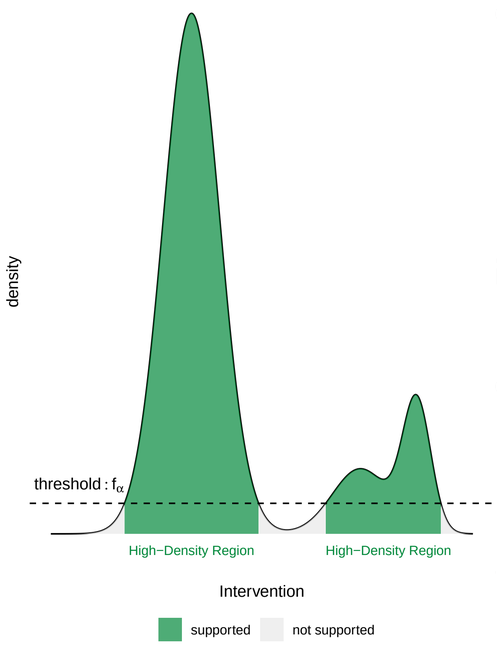

Figure 1: High-Density Regions

Step 2: Find most feasible intervention

For a given oracle intervention target $a \in \mathcal{A}$, **a data-adaptive intervention** can be outlined as:

$$\begin{cases} \text{Intervene with } a, & \text{if supported,} \\ \text{Intervene } \textbf{adaptively}, & \text{otherwise.} \end{cases}$$

Formally we define **the most feasible intervention** as follows,

$$\mathrm{d}(a, \mathbf{l}; \mathcal{D}) = \begin{cases} a, & \text{if } a \in \mathcal{A}_\alpha(\mathbf{l}), \\ h(a, \mathbf{l}; \mathcal{D}), & \text{otherwise.} \end{cases}$$

$$h(a, \mathbf{l}; \mathcal{D}) = \underset{a^* : a^* \in \mathcal{A}_\alpha(\mathbf{l})}{\operatorname{argmin}} |a^* - a|$$

The corresponding causal estimand is:

$$m^{\text{feasible}} : \mathcal{A} \to \mathcal{Y}; a \mapsto E(Y^{a^{\mathrm{d}}});$$
$$\text{or } a \mapsto P(a \in \mathcal{A}_\alpha) \cdot E(Y^a | a \in \mathcal{A}_\alpha) + (1 - P(a \in \mathcal{A}_\alpha)) \cdot E(Y^{h(a,\mathbf{l};\mathcal{D})} | a \notin \mathcal{A}_\alpha).$$

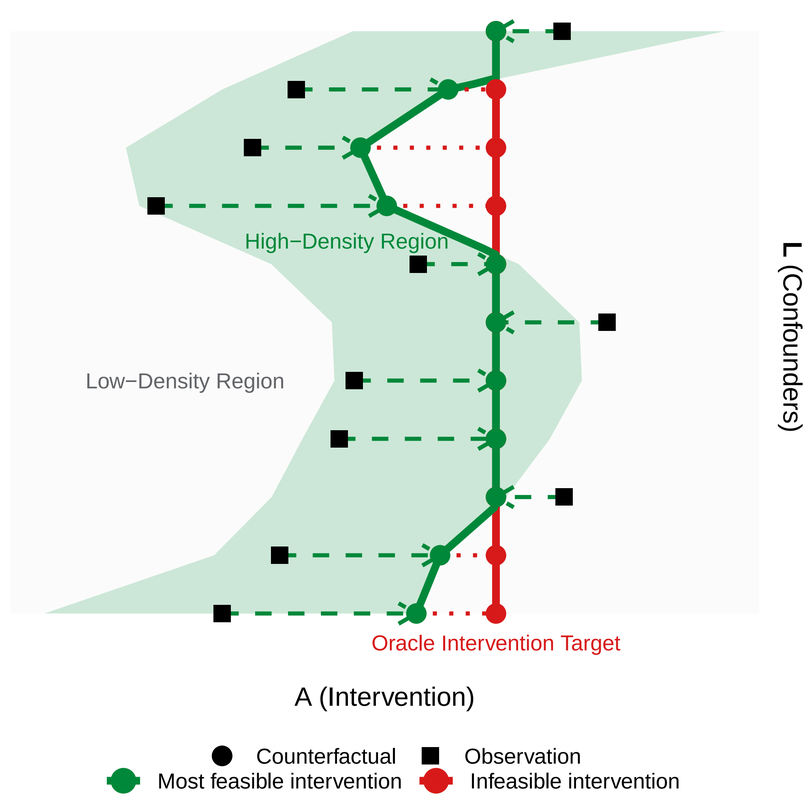

Figure 2: Most Feasible Intervention

Notations: $Y/\mathcal{Y}$: Outcome (space); $A/\mathcal{A}$: Intervention (space); $\mathbf{L}/\mathcal{L}$: Confounders (space); $f(a|\mathbf{L})$: Conditional density function of A given $\mathbf{L}$;

Data & Results

Our simulation studies show that the most feasible intervention reduces absolute bias compared to standard and trimming approaches under positivity violations (Figure 3). In regions where positivity violations are absent, the proposed approach can recover the standard estimand.
Using data from the CHAPAS-3 trial of HIV-positive children in Zambia and Uganda [3, 4], we examine how counterfactual viral failure probabilities vary with efavirenz concentrations (EFV, mg/L) at $t = 36$ weeks. Non-overlap ratios (Figure 4) are low within the central EFV range (1–3.5 mg/L), enabling reliable causal estimation. At boundaries (< 1 or > 3.5 mg/L), ratios rise, indicating strong positivity violations, with sharp increases near 0 mg/L. Figure 5 shows causal curves across coverage levels; higher coverage level tolerates more violations, while instability in trimming estimands arises under severe violations.

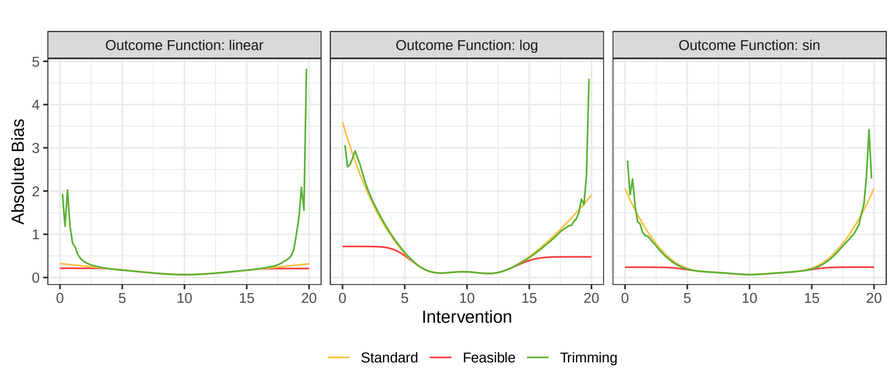

Figure 3: Absolute Bias of Concentration-Response Curves

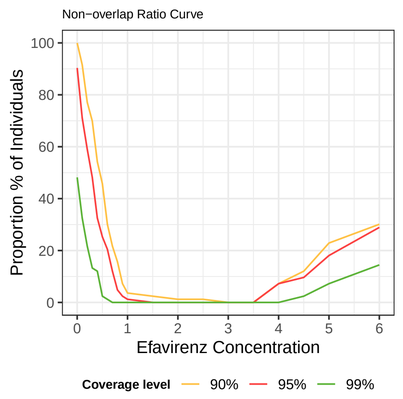

Figure 4: The Non-Overlap Ratio

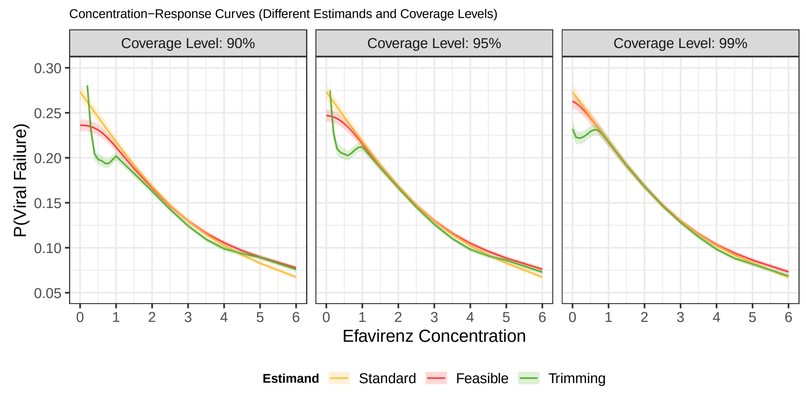

Figure 5: Estimated Concentration-Response Curves

Conclusions

The proposed diagnostic measure provides an effective tool for identifying positivity violations. The central idea of the intervention strategy is to adhere as closely as possible to the true causal dose-response curve, deviating only when the oracle target intervention is infeasible based on the available data. In such cases, the strategy substitutes the intervention with the most feasible alternative, maintaining interpretability and mitigating potential bias.

References

[1] Michael Schomaker et al. "Causal Inference for Continuous Multiple Time Point Interventions". In: *Statistics in Medicine* (2024).
[2] Iván Díaz et al. "Nonparametric causal effects based on longitudinal modified treatment policies". In: *Journal of the American Statistical Association* (2023).
[3] Veronica Mulenga et al. "Abacavir, zidovudine, or stavudine as paediatric tablets for African HIV-infected children (CHAPAS-3): an open-label, parallel-group, randomised controlled trial". In: *The Lancet Infectious diseases* (2016).
[4] G. Abongomera et al. "Improved Adherence to Antiretroviral Therapy Observed Among HIV-Infected Children Whose Caregivers had Positive Beliefs in Medicine in Sub-Saharan Africa". In: *Aids and Behavior* (2017).

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Working paper available on arXiv: